

# Second harmonic electromagnetic emission of a turbulent magnetized plasma driven by a powerful electron beam

I.V. Timofeev

*Budker Institute of Nuclear Physics SB RAS, 630090, Novosibirsk, Russia  
Novosibirsk State University, 630090, Novosibirsk, Russia*

The power of second harmonic electromagnetic emission is calculated for the case when strong plasma turbulence is excited by a powerful electron beam in a magnetized plasma. It is shown that the simple analytical model of strong plasma turbulence with the assumption of a constant pump power is able to explain experimentally observed bursts of electromagnetic radiation as a consequence of separate collapse events. It is also found that the electromagnetic emission power calculated for three-wave interaction processes occurring in the long-wavelength part of turbulent spectrum is in order-of-magnitude agreement with experimental results.

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Electromagnetic emission of a turbulent plasma at the fundamental plasma frequency  $\omega_p$  and its harmonics has been the subject of active theoretical and experimental research for several decades. This radiation carries information about properties of plasma turbulence, and registration of plasma emission is one of the most efficient ways of studying physical processes occurring in space plasmas. That is why in most papers the problem of turbulent plasma emission is considered in the context of space phenomena such as type III solar radio bursts [1–7] and emissions of planet’s magnetospheres [8, 9]. Our interest to this problem is motivated by laboratory experiments [10], in which electromagnetic radiation at the doubled plasma frequency is generated during injection of a high-current relativistic electron beam into the plasma confined in the GOL-3 multimirror trap. In contrast to previous beam-plasma experiments [11–14], we study emission properties of rather hot ( $T = 1\text{--}2$  keV) plasmas in sufficiently strong magnetic fields ( $\Omega = \omega_c/\omega_p \sim 1$ , where  $\omega_c$  is the electron cyclotron frequency).

Several generation mechanisms of second harmonic plasma emission exist. In the context of type III radio bursts, either weakly turbulent coalescence of Langmuir waves  $\ell + \ell \rightarrow t$  [15, 16], or generation of electromagnetic waves by collapsing caverns in strong plasma turbulence [5, 17, 18] are discussed. Moreover, such electromagnetic waves can be produced due to the Langmuir harmonic waves [19] scattering off density fluctuations. It is obvious that the power of electromagnetic emission depends essentially on what nonlinear processes form the turbulent spectrum. In the theory of weak turbulence the important role is played by the electrostatic Langmuir decay  $\ell \rightarrow \ell' + s$  ( $s$  denotes ion-acoustic waves) and the reverse process  $\ell + s \rightarrow \ell'$ . In an optically thick plasma, formation of a turbulent spectrum can be also affected by nonlinear processes  $\ell \rightarrow t + s$  and  $\ell + s \rightarrow t$  involving electromagnetic waves with the plasma frequency  $\omega_p$ . In models of strong plasma turbulence [21–23] wave energy transfers through the spectrum due to Langmuir wave scattering off density fluctuations and collapse of localized Langmuir wave packets. Alternative models of

strong plasma turbulence, in which wave collapse is suppressed by either the direct conversion of Langmuir waves to damping modes [24] or radiative losses [25], are also discussed.

Experiments on turbulent plasma heating at the GOL-3 multimirror trap show that intensity of electromagnetic emission demonstrates not only smooth variation in time, but also bursts with the duration of 2–10 ns, which we tend to associate with separate collapse events. Thus, the model of strong plasma turbulence, proposed in Ref. [21] and verified later by numerical simulations [26], seems most appropriate for our experiments. In Ref. [3] this analytic model is used to estimate the power of electromagnetic emission in the problem of type III radio bursts. In order to explain the results of laboratory beam-plasma experiments we will modify the model of Ref. [26] by taking into account saturation of the pumping power due to beam trapping and generalize the calculation procedure of the second harmonic emission power of Ref. [3] to an arbitrary magnetic field.

According to the model of strong plasma turbulence [21, 26] an isotropic turbulent spectrum of an unmagnetized plasma can be divided into three typical regions: source region, inertial range and dissipation region. The source region occupies small wavenumbers  $k < k_M \simeq \sqrt{W/(nT)}/r_D$  ( $r_D$  is the Debye length) and consists of untrapped Langmuir waves, which are the products of beam-driven Langmuir waves scattering off long-wavelength density fluctuations. It is assumed that the spectral density of wave energy inside this part of the spectrum is independent on  $k$ . In the inertial range, modulation instability results in trapping of Langmuir waves in local density wells and is followed by the wave collapse, which is responsible for the formation of the power-law spectrum. In the small-wavelength region various dissipation mechanisms come into force and spectral wave energy decreases rapidly with the increase of  $k$ . Thus, second harmonic plasma emission can be generated (i) due to Langmuir wave coalescence in the source region containing most of the wave energy and (ii) due to the collapse of trapped plasma oscillations which, despite the

low energy content, reach high energy densities at the late stage of collapse and result in radiation bursts.

Let us calculate the level of the wave energy density  $W$  in a plasma turbulence that is pumped by an energy source with the power  $P$ , and estimate the typical duration of radiation bursts produced in separate collapse events. Energy balance between different parts of turbulent spectrum can be written in the form

$$P \approx \omega_p \sqrt{\frac{\langle \delta n^2 \rangle}{n^2}} W_R \approx \lambda(W) \omega_p \sqrt{\frac{m_e}{m_i} \frac{W}{nT}} W. \quad (1)$$

The first equation describes the balance between pumping of beam-excited resonant waves with the energy density  $W_R$  and dissipation produced by their scattering off long-wavelength density fluctuations. The rms level of these fluctuations is determined by the wave energy density  $W$  concentrated in the source region:  $\sqrt{\langle \delta n^2 \rangle}/n = \alpha W/nT$ , where  $\alpha = 0.7$  is the numerical coefficient obtained in 2D simulations. The second equation in (1) shows that the power that comes to the source region from the pump is balanced by the power that leaves this region due to the wave collapse. Here, according to Ref. [26], the rate of collapse is determined not only by the rate of modulation instability, but also by the factor  $\lambda(W) = 2\lambda(W/nT)^{1/2}$  accounting for finite time that takes the collapse to reach the self-similar regime ( $\lambda \simeq 0.7$ ).

The pump power is usually estimated as  $P = 2\Gamma W_R$ , where  $\Gamma$  is the linear growth rate of the beam-plasma instability. In our experiments, however, the powerful electron beam relaxes in the so-called trapping regime [27], for which the energy pump to the beam-excited coherent wave packets is saturated by the beam nonlinearity. Indeed, 1D particle-in-cell simulations [28] show that in our case the pump power does not depend on evolving parameters of the plasma turbulence. That is why in the balance equation this power can be considered as a given constant  $P = \text{const}$ . We can estimate this value from experimental data:  $P = \beta n T_0 / \tau_0 = 100 \text{ kW/cm}^3$ , assuming that it takes  $\tau_0 = 3 \mu\text{s}$  to heat the plasma with the density  $n = 2 \cdot 10^{14} \text{ cm}^{-3}$  up to the electron temperature  $T_0 = 1 \text{ keV}$ . The factor  $\beta = 6$  takes into account that most of the dissipated wave energy goes to the formation of high-energy tails.

For the resonant  $W_R$  and nonresonant  $W$  wave energies in this model we get

$$\frac{W}{nT} \approx \frac{1}{\sqrt{2\lambda}} \left( \frac{m_i}{m_e} \right)^{1/4} \left( \frac{P}{\omega_p nT} \right)^{1/2}, \quad (2)$$

$$\frac{W_R}{W} \approx \frac{2\lambda}{\alpha} \sqrt{\frac{m_e}{m_i}}. \quad (3)$$

The characteristic duration of wave collapse can be estimated as

$$\tau_c \sim \frac{1}{2\lambda\omega_p} \sqrt{\frac{m_i}{m_e} \frac{nT}{W}}.$$

For typical parameters of beam-plasma experiments at the GOL-3 multimirror trap, the wave energy density reaches the value  $W/nT = 0.01$  for the electron temperature  $T = 1 \text{ keV}$ . The collapse duration at the same stage appears to be 3-4 ns, which is in a good agreement with the duration of radiation bursts observed experimentally. Thus, experimental data do not contradict the basic ideas of our model, and we can use estimates for the wave energy density and the characteristic width of the energy-containing region of plasma turbulence to calculate the emission power. We also assume that these estimates are not extremely sensitive to the external magnetic field.

Let us calculate the power of second harmonic plasma emission which is generated due to coalescence of untrapped Langmuir waves  $\ell + \ell \rightarrow t$  in the long-wavelength part of turbulent spectrum  $k < k_M$ . By Langmuir waves in the cold magnetized plasma we mean plasma oscillations pertaining to the upper-hybrid branch. Nonlinear interaction of such waves can be described in the framework of weak turbulence, but in contrast to the standard theory we will take into account model damping of two-time correlation functions, which is used to describe the effect of finite life-time of Langmuir plasmons due to their scattering off density fluctuations. Let us represent the electric field in the following form

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma} \int \mathbf{e}_k^{\sigma} E_k^{\sigma}(t) e^{i\mathbf{k}\mathbf{r} - i\omega_k^{\sigma} t} d^3k,$$

where  $\omega_k^{\sigma}$  and  $\mathbf{e}_k^{\sigma}$  denote eigenfrequencies and eigenvectors of linear plasma modes,  $E_k^{\sigma}$  — their slowly varying amplitudes, and  $\sigma$  indicates the branch to which they belong. In the general case, three-wave interaction  $\sigma' + \sigma'' \rightarrow \sigma$  is described by the equation

$$\frac{\partial E_k^{\sigma}}{\partial t} = -\frac{4\pi i e i \omega_k^{\sigma} t}{(\partial \Lambda^{\sigma} / \partial \omega) \omega_k^{\sigma}} \frac{\partial}{\partial t} \left( \mathbf{e}_k^{*\sigma} \cdot \mathbf{j}_k^{(2)}(t) \right), \quad (4)$$

where

$$\Lambda^{\sigma}(\mathbf{k}, \omega) = |\mathbf{k} \cdot \mathbf{e}_k^{\sigma}|^2 c^2 - k^2 c^2 + \omega^2 \mathbf{e}_k^{*\sigma} \hat{\varepsilon}(\mathbf{k}, \omega) \mathbf{e}_k^{\sigma},$$

$\hat{\varepsilon}(\mathbf{k}, \omega)$  is the dielectric tensor and  $\mathbf{j}_k^{(2)}$  is the Fourier transform of the second-order nonlinear electron current, which in the cold plasma limit takes the form

$$\left( \mathbf{j}_k^{(2)} \cdot \mathbf{e}_k^{*\sigma} \right) = \frac{en}{(2\pi)^{3/2}} \int \frac{e^2 E_{k_1}^{\sigma'} E_{k_2}^{\sigma''}}{m^2 \omega_{k_1}^{\sigma'} \omega_{k_2}^{\sigma''}} G_{k, k_1, k_2}^{\sigma \sigma' \sigma'} \times \frac{e^{-i\omega_+ t}}{\omega_+} \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) d^3k_1 d^3k_2, \quad (5)$$

$$G_{k, k_1, k_2}^{\sigma \sigma' \sigma'} = \omega_+ T_{k_2, \alpha \beta}^{\sigma''} e_{k_2, \beta}^{\sigma''} T_{k_1, ij}^{\sigma'} e_{k_1, j}^{\sigma'} \times \left( \frac{k_{1i}}{\omega_{k_1}^{\sigma'}} e_{k, \alpha}^{*\sigma} + \frac{k_{2\alpha}}{\omega_{k_2}^{\sigma''}} e_{k, i}^{*\sigma} \right) + e_{k, \alpha}^{*\sigma} T_{\alpha \beta}^{(+)} g_{\beta}, \quad (6)$$

$$g_{\alpha} = k_{2\alpha} T_{k_1, ij}^{\sigma'} e_{k_2, i}^{\sigma''} e_{k_1, j}^{\sigma'} + k_{2i} T_{k_1, ij}^{\sigma'} e_{k_1, j}^{\sigma'} \times \left[ T_{k_2, \alpha \beta}^{\sigma''} e_{k_2, \beta}^{\sigma''} - \left( 1 - \frac{\Omega^2}{(\omega_{k_2}^{\sigma''})^2} \right) e_{k_2, \alpha}^{\sigma''} \right] +$$

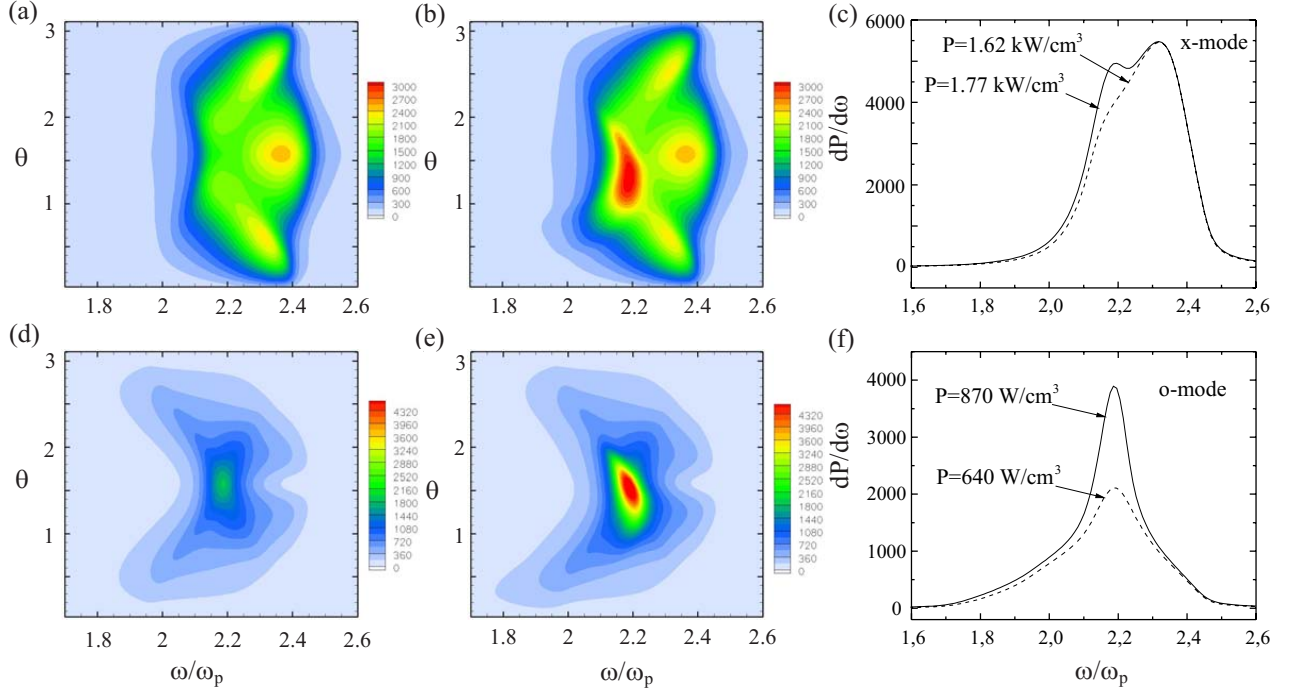


Figure 1. The spectral power  $dP/(d\omega d\theta)$  of x-mode emission for the isotropic turbulent spectrum (a) and for the spectrum including resonant waves (b). The spectral power  $dP/(d\omega d\theta)$  of o-mode emission for the isotropic turbulent spectrum (d) and for the spectrum including resonant waves (e). The spectral power  $dP/d\omega$  of x-mode (c) and o-mode (f) emission for the isotropic (dashed line) and anisotropic (solid line) spectrum.

$$(k_1, \sigma' \leftrightarrow k_2, \sigma''), \quad (7)$$

$$T_{k, \alpha\beta}^\sigma = \frac{1}{1 - \frac{\Omega^2}{(\omega_k^\sigma)^2}} \left[ \delta_{\alpha\beta} - i \frac{\Omega}{\omega_k^\sigma} e_{\alpha\beta\gamma} h_\gamma - \frac{\Omega^2}{(\omega_k^\sigma)^2} h_\alpha h_\beta \right],$$

$$\omega_+ = \omega_{k_1}^{\sigma'} + \omega_{k_2}^{\sigma''},$$

In dimensionless units  $\omega_p t$ ,  $\omega/\omega_p$ ,  $x\omega_p/c$ ,  $kc/\omega_p$ ,  $eE_k^\sigma(\omega_p/c)^3/(mc\omega_p)$  for time, frequency, position, wavenumber and electric field amplitude of a plasma mode, respectively, three-wave interaction processes  $\ell + \ell \rightarrow t$  between Langmuir and electromagnetic waves are described by the equation

$$\frac{\partial E_k^t}{\partial t} = -\frac{1}{2(2\pi)^{3/2}(\partial\Lambda^\ell/\partial\omega)_{\omega_k^t}} \int \frac{E_{k_1}^\ell E_{k_2}^\ell}{\omega_{k_1}^\ell \omega_{k_2}^\ell} G_{k, k_1, k_2}^{t\ell\ell} \times e^{i(\omega_k^t - \omega_{k_1}^\ell - \omega_{k_2}^\ell)t} \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) d^3k_1 d^3k_2. \quad (8)$$

According to Ref. [3] we assume that Langmuir waves, scattering off long-wavelength density fluctuations, change their phases stochastically with the characteristic frequency  $\nu/\omega_p = W^\ell/nT$ . In this case, the temporal correlation function of Langmuir electric fields can be written in the form

$$\langle E_k^\ell(t) E_q^{*\ell}(t') \rangle = I_k^\ell \delta(\mathbf{k} - \mathbf{q}) e^{-\nu|t - t'|}. \quad (9)$$

For the average energy of Langmuir turbulence we get

$$\frac{W^\ell}{nmc^2} = \int W_k^\ell d^3k, \quad (10)$$

$$W_k^\ell = \frac{1}{2(2\pi)^3 \omega_k^\ell} \left( \frac{\partial \Lambda^\ell}{\partial \omega} \right)_{\omega_k^\ell} I_k^\ell.$$

The spectral energy density of electromagnetic waves produced in spontaneous processes  $\ell + \ell \rightarrow t$  is governed by the equation

$$\frac{\partial W_k^t}{\partial t} = \frac{2\pi}{\omega_k^t (\partial\Lambda^t/\partial\omega)_{\omega_k^t}} \times \int \frac{W_{k_1}^\ell W_{k_2}^\ell |G_{k, k_1, k_2}^{t\ell\ell}|^2 \Delta_{k, k_1, k_2} \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)}{\omega_{k_1}^\ell (\partial\Lambda^\ell/\partial\omega)_{\omega_{k_1}^\ell} \omega_{k_2}^\ell (\partial\Lambda^\ell/\partial\omega)_{\omega_{k_2}^\ell}} d^3k_1 d^3k_2, \quad (11)$$

where  $\Delta_{k, k_1, k_2}$  is the function describing correlation broadening of the resonance  $\omega_k^t - \omega_{k_1}^\ell - \omega_{k_2}^\ell = 0$ :

$$\Delta_{k, k_1, k_2} = \frac{2\nu/\pi}{(\omega_k^t - \omega_{k_1}^\ell - \omega_{k_2}^\ell)^2 + 4\nu^2}.$$

Thus, in the case of azimuthally symmetric turbulence, the spectral power of second harmonic electromagnetic

emission in units of  $nmc^2$  is given by the integral

$$\frac{dP}{d\omega} = 2\pi \int_0^\pi \sin\theta d\theta \left( \frac{k^2}{d\omega/dk} \frac{\partial W_k^t}{\partial t} \right)_{k(\omega)}, \quad (12)$$

where  $k(\omega)$  is the solution of  $\omega = \omega_k^t$  and  $\theta$  is the polar angle of  $\mathbf{k}$ .

Let us compute the spectral power of electromagnetic emission for parameters typical for beam-plasma experiments in the GOL-3 multimirror trap. In the regime with the plasma density  $n = 2 \cdot 10^{14} \text{ cm}^{-3}$ , the external magnetic field  $\Omega = 0.8$  and the electron temperature  $T = 1 - 2 \text{ keV}$ , the power of electromagnetic emission was estimated experimentally as  $0.1 \div 1 \text{ kW/cm}^3$ . In our theoretical model, the isotropic part of long-wavelength plasma turbulence should occupy the spectral region  $k \in (0.1; 2.45)\omega_p/c$  and should contain the energy  $W^\ell/nT = 0.01$  for the typical electron temperature  $T = 1 \text{ keV}$ . Moreover, in the beam-excited turbulence there is an anisotropic population of resonant Langmuir waves containing about 5% of turbulence energy. This part of energy is concentrated in a rather small spectral region:  $k \in (1.1; 1.3)$  and  $\theta \in (0; 0.3)$ . The computation results for the emission power of ordinary (o-mode) and extraordinary (x-mode) electromagnetic waves are presented in Fig. 1. To analyze the contribution of resonant waves to plasma emission, we also present computations accounting for the isotropic part of turbulence only.

The angular distribution of the emission power  $dP/d\omega d\theta$  shows that both x-mode [Fig. 1 (a), (b)] and o-mode [Fig. 1 (d), (e)] are radiated predominantly in the transverse to the magnetic field direction. It is also shown in Fig. 1(c) and 1(f) that the total emission power integrated over angle and frequency reaches the value of  $2 \text{ kW/cm}^3$  and is dominated by the x-mode contribution. One can see that resonant waves do not substantially affect x-mode emission and result in the significant increase of the o-mode emission power. From the emission spectrum it is also seen that the main role in generation of electromagnetic waves is played by almost potential Langmuir waves with frequencies  $\omega > \omega_p$ .

In conclusion, we calculate second harmonic electromagnetic emission of a turbulent magnetized plasma driven by a powerful electron beam. We found that the simple analytical model of strong plasma turbulence with the assumption of constant pump power explains the results of laboratory beam-plasma experiments at the GOL-3 multimirror trap. We show that the typical duration of electromagnetic bursts observed in these experiments is in a good agreement with the theoretical estimate of collapse duration. Our theory does also predict by order of magnitude experimental results for the total emission power and explains polarization of this emission.

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